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# Arbitrary amplitude ion acoustic solitary waves in moderately relativistic degenerate plasma

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## Abstract

Arbitrary amplitude solitary waves are investigated in weakly relativistic degenerate quantum plasma containing electrons and ions by employing Sagdeev's pseudopotential approach. It is found that relativistic degeneracy, ion-to electron Fermi temperature ratio and quantum diffraction have significant contributions in determining the nature of pseudopotential well as well as the formation and properties of ion acoustic solitary wave structures in a two-component electron-ion dense quantum plasma.

**Keywords-** quantum plasmas, quantum hydrodynamic model, relativistic degeneracy, ion-acoustic waves, Sagdeev's pseudopotential approach, solitary waves.

## I. INTRODUCTION

The quantum plasma physics has a long and diverse tradition [1-3]. Recently there has been a surge in the research interest in the field [4]. Quantum plasma is characterize by low temperature and high density. In order to associate a meaning to the expression low temperature, it is important to compare the thermal interaction energies of the system. The coupling which determines whether a system is quantum or classical is the ratio of the potential energy to the kinetic energy. The expression for quantum and classical kinetic energies given below. For both classical and quantum Coulomb systems, the mean interaction energy $U_{pot}$ is the same and is equal to:

$$U_{pot} = \frac{e^2 n_0^{1/3}}{\varepsilon_0} \qquad (1)$$

But the mean kinetic energies ($K_c$ for classical and $K_Q$ for quantum) are not same for both the systems. They are given by the following relations:

$$K_C = k_B T \qquad (2)$$

$$K_Q = k_B T_F \qquad (3)$$

Here $T_F$ is the Fermi temperature given by:

$$T_F = \frac{\hbar^2}{2m} \cdot \frac{(3\pi^2 n)^{2/3}}{k_B} \qquad (4)$$

The classical and quantum coupling parameters are given by:

$$\Gamma_C = \frac{U_{pot}}{K_C} = \frac{e^2 n_0^{\frac{1}{3}}}{\varepsilon_0 k_B T} \approx \left(\frac{1}{n_0 \cdot \lambda_D{}^3}\right)^{\frac{1}{3}} = 2.1\times 10^{-7} \times \frac{n_0^{\frac{1}{3}}}{T} \qquad (5)$$

$$\Gamma_Q = \frac{U_{pot}}{K_Q} = \frac{2me^2\pi^{-4/3}}{\pi^{4/3}\varepsilon_0 \hbar^2 n_0^{1/3}} \approx \left(\frac{1}{n_0 \lambda_F{}^3}\right)^{\frac{2}{3}} = \frac{10^{11}}{2} n_0^{-\frac{1}{3}} \qquad (6)$$

From the above equation we can say the classical weakly coupled plasma are found to be dilute and the quantum weakly coupled plasmas are found to be dense. The degeneracy parameter $\chi = T_F / T$ determines whether the system is quantum or classical according to which the Wigner formulation or the Vlasov formulation will be used. So far there has been a few mathematical models that describe the properties of quantum plasma. Some of them are the Schrödinger-Poisson (SP) formulation and Wigner-Poisson (WP) formulation. The SP model describes the hydrodynamic behavior of plasma particles in quantum scales whereas the WP model is often used in the study of quantum kinetic behavior of plasma.

With the recent development of quantum plasma physics contributions of Haas [5], Manfredi [6], Shukla [4], Bonitz [7], Brodin [8], Marklund [9], Eliasson [10], Ghosh [11, 12], Chandra [13-24], and others and others [25, 26] have strengthen the field. All of them have employed the quantum hydrodynamic model (QSD) generalizes the classical fluid model for plasmas with the inclusion of a quantum correction term known as Bohm potential. The QSD model incorporates quantum statistical effects through the equation of state. In many cases these equation of state has been incorporated from Fermi Dirac distribution considering the plasma particles as quantum elements. Some have also considered degeneracy pressure arising out of relativistic effects in degenerate plasmas [27].

It was found in the early years of plasma physics that the underline physics of non-linear quantum like equation can be better understood by rewriting those equations in the form of hydrodynamic equations which essentially represents the densities and momentum evaluation of quantum particles. It was an elegant treatment by Bohm [2] and Madelung [28] by introducing and eikonal representation for the wave function evolution in the non-stationary Schrödinger equations. The Madelung equations for quantum fluid is derived by [29-35]. In order to incorporate quantum fluid formalism, the quantum electron fluid equations were derived for the Klein-Gordon equations [36] and for the Dirac equations [37-39]. Over the past few years there has been an increase in the research of quantum plasmas as it finds applications in astrophysical plasma (white dwarf, neutron star etc) [40-46]. In laboratory produce plasmas, quantum effects are found in metal nanostructures, bio-photonics, CNT, microelectronics, laser solid interaction etc. Modulational instability, solitary structures has been studied by many authors by using reductive perturbation technique (RPT). It can only be used for small amplitude waves. Large amplitude waves can be investigated only by an exact mathematical tour, such as a pseudo potential approach.

Ion acoustic solitary waves (IASWs) have been studied by many authors [47-55] (A 25-30,34-36). Non-linear IAWs in collision less unmagnetised quantum plasma has been investigated by Haas *et al.* [47] in one dimension. He incorporated the Bohm potential and quantum statistical pressure using Fermi-Dirac distribution. Misra and Bhowmik [48] studied the nonlinear IAWs in quantum plasma in non-planar geometry by solving the Kodomstev-Petviasville (KP) equation. Moslem et al. have solved the Zakhrov-Kuznetsov (ZK) equation in quantum magnetoplasma. Some authors have also investigated dust IAWs and its behaviors. Masood et al. [52] have studied the solitary structure by Korteweg-de Vries (KDV) equation. Majority of these works are done using reductive perturbation technique (RPT) which a valid for small amplitude waves in order to investigate large amplitude solitary structure. It is necessary to analyse the exact solitary structure in which the total nonlinearity of the system is considered without any approximation. In plasma Sagdeev's psedopotential is one such method which is widely used in various plasma models [53-55]. Ion acoustic solitary waves in unmagnetised electron plasma has been studied by Masood & Mushtaq [56] using Sagdeev's pseudopotential approach. In their investigation they have neglected the ion temp which has finite effect in the formation and properties of ion acoustic solitary waves. The findings of Ali *et al* [57] have given much more insight to the properties of IAWs. By neglecting the effect of ion temperature. In ultra-dense matter the presence of relativistic degeneracy effects is given by Chandrasekhar in 1939 [27]. The expression given by him has been applied in quantum plasmas by Akbari-Moghanjoughi [58], Labany *et al.* [59], Mamun and Shukla [60], Chandra *et al*. [17, 18]. Most of the work in ion acoustic waves including relativistic degeneracy effects were limited to small amplitude waves for which reductive perturbation technique is generally used. Large amplitude waves in such a relativistic quantum plasma have so far to our knowledge has not been investigated. The motivation of the present paper is to study the large amplitude ion acoustic solitary structure weakly relativistic degenerate electron ion plasma by employing Sagdeev's psedopotential method.

The paper is organized in the following way; in section two, the basic dynamic equations of the system using QHD model are introduced with proper justifications. It also derives the expression for pseudopotential U(n) in terms of n. The third section investigates the solitary properties of ion acoustic wave and it's dependence of different parameters. Finally we discuss our results and came to a conclusion.

## II. BASIC FORMULATIONS

Let us consider the homogeneous and unmagnetised electron-ion quantum plasma. In quantum plasma the effect of ion temperature on the IAW is studied by assuming the electrons to be inertialess & the ions are taken to be inertial. The phase velocity of the wave is taken to be $V_{Fi} << \omega / k << V_{Fe}$ (where $V_{Fi}$ and $V_{Fe}$ are the Fermi velocities of ions and electrons respectively). Ion pressure effects due to ion Fermi temperature can therefore be ignored. The basic dynamic equations ignoring the non-linear mechanisms of ion acoustic waves in quantum plasmas are given in the dimensional (unnormalised) form as:

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_e)}{\partial x} = 0 \tag{7}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0 \tag{8}$$

$$0 = \frac{e}{m_e}\frac{\partial \phi}{\partial x} - \frac{1}{m_e n_e}\frac{\partial p_e}{\partial x} + \frac{h}{2m_e^2}^{2}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_e} \big/ \partial x^2}{\sqrt{n_e}}\right) \tag{9}$$

$$\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x} - \sigma_1 n_i \frac{\partial n_i}{\partial x} \tag{10}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e(n_i - n_e) \tag{11}$$

Here $n_j$, $v_j$ , $m_j$ ,$-e$ are the density, velocity field, mass, and charge, respectively where j =*e*, *i* stands for electrons and ions. Meanwhile, ħ=*h*/2π is the reduced Planck constant, $\phi$ is the electrostatic wave potential, $p_e$ is the electron pressure, and $\sigma_1 = 3[T_{Ei} / T_{Fe}]$ is the ion-to-electron Fermi Temperature ratio, where $T_{Fj}$ is the Fermi temperature of the j$^{th}$ species. At equilibrium, we have $n_{i0}=n_{e0}=n_0$. The equation of state for ions is given by considering them to be a fully degenerate 1-D Fermi gas [47]. Following Chandrasekhar (1939) the electron degeneracy pressure in fully degenerate and relativistic configuration can be expressed in the following form:

$$P_e = \left(\frac{\pi m_e^4 c^5}{3h^3}\right)[R_e(2R_e^2 - 3)\sqrt{1+R_e^2} + 3\sinh^{-1} R] \qquad (12)$$

in which $R_j = p_{Fj}/m_e c = \left[3h^3 n_j / 8\pi m_e^3 c^3\right]^{\frac{1}{3}} = R_{j0} n_j^{\frac{1}{3}}$

[ $R_{j0} = \left(n_{j0}/n_0\right)^{1/3}$; $n_0 = 8\pi m_e^3 c^3 / 3h^3 \approx 5.9\times 10^{29}\, cm^{-3}$ ], 'c' being the speed of light in vacuum. $p_{Fj}$ is the electron Fermi relativistic momentum. It is to be noted that in the limits of very small values of relativity parameter $R_e$ (the weakly relativistic case) we obtain:

$$P_e = \frac{1}{20}\left(\frac{3}{\pi}\right)^{\frac{2}{3}} \frac{h^2}{m_e} n_e^{\frac{5}{3}} \text{ (For } R_e \to 0\text{)} \qquad (13)$$

Note that the degenerate electron pressure depends only on the electron number density but not on the electron temperature.

Now, let $p_e = \frac{1}{20}\left(\frac{3}{\pi}\right)^{\frac{2}{3}} \frac{h^2}{m_e} n_e^{\frac{5}{3}} = F n^{\frac{5}{3}}$ (14)

where $F = \frac{1}{20}\left(\frac{3}{\pi}\right)^{\frac{2}{3}} \frac{h^2}{m_e}$

So; $\frac{\partial p_e}{\partial x} = \frac{5F}{3} n_e^{2/3} \frac{\partial n_e}{\partial x}$

$$\frac{1}{n_e}\frac{\partial p_e}{\partial x} = \frac{5F}{3} n_e^{-1/3} \frac{\partial n_e}{\partial x} \qquad (15)$$

Now putting (14) in the equation (9) and using the following normalization:

$x \to \frac{x\omega_i}{c_s},\ t \to t\omega_i,\ \phi \to \frac{e\phi}{2k_B T_{Fe}}, n_j \to \frac{n_j}{n_0},\ u_j \to \frac{u_j}{c_s}$ the set of normalized (dimensionless) equations are given as:

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_e)}{\partial x} = 0 \qquad (16)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0 \qquad (17)$$

$$0 = \frac{\partial \phi}{\partial x} - \frac{5F}{3} n_e^{-1/3} \frac{\partial n_e}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_e} / \partial x^2}{\sqrt{n_e}}\right) \qquad (18)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{\partial \phi}{\partial x} - \sigma_1 n_i \frac{\partial n_i}{\partial x} \qquad (19)$$

$$\frac{\partial^2 \phi}{\partial x^2} = (n_i - n_e) \qquad (20)$$

in which $\omega_e = \sqrt{4\pi n_{e0} e^2 / m_e}$ is the plasma frequency, $c_s = \sqrt{2k_B T_{Fe} / m_e}$ is the quantum ion-acoustic speed. H is the non-dimensional quantum diffraction parameter defined as $H = \hbar\omega_{ec} / 2k_B T_{Feh}$, where $T_{Fe}$ is the Fermi temperatures for electrons.

In order to get localize stationary solution, let us assume that all dependent variables are functions of single independent variable:

$$\xi = x - Mt \qquad (21)$$

where M is the Mach number defined by $v/c_s$, v is the velocity of the nonlinear waveform moving with the frame. By integrating (18) once and applying boundary conditions $n_e \to 1$ & $\phi \to 0$ at $\xi = |\pm\infty|$; we obtain:

$$\phi = -\frac{5F}{2} + \frac{5F}{2} n_e^{2/3} - \frac{H^2}{2}\frac{1}{\sqrt{n_e}}\frac{\partial^2 \sqrt{n_e}}{\partial x^2} \qquad (22)$$

From the ion continuity equation (17) and ion momentum equation (19) with proper boundary conditions $\phi \to 0,\ v_i \to 0,\ n_i \to 1\ as\ \xi \to |\pm\infty|$ we obtain:

$$v_i = M(n_i - 1)/n_i \qquad (23)$$

$$v_i^2 - 2Mv_i + \sigma_1 n_i^2 - \sigma_1 = -2\phi \qquad (24)$$

Substituting equation (23) in (24) we get,

$$\phi = \frac{M^2}{2}[1 - \frac{1}{n_i^2}] + \frac{\sigma_1}{2}(1 - n_i^2) \qquad (25)$$

Now by employing quasi-neutrality conditions

$$n_i \approx n_e = n \qquad (26)$$

and also substituting $z = \sqrt{n}$ (27)

from equations (22) - (25) we obtain

$$\frac{H^2}{2}\frac{\partial^2 Z}{\partial \xi^2} = -\frac{5F}{2} Z + \frac{5F}{2} Z^{7/3} - \frac{M^2}{2}[Z - \frac{1}{Z^3}] - \frac{\sigma_1}{2}[Z - Z^5] \qquad (28)$$

Multiplying both sides of equation (28) by $dZ/d\xi$ and integrating with the boundary condition $n'' \to 0$ and $n' \to 0$ and $n \to 0$, (where primes represent derivatives with respect to ξ) we obtain the nonlinear differential equation in terms of density as:

$$\frac{1}{2}\left(\frac{dn}{d\xi}\right)+u(n)=0 \qquad (29)$$

Where, the Sagdeev's pseudopotential is defined as equation:

$$U(n)=\frac{8n}{H^2}\left[\begin{array}{c}\left(5F+M^2+\sigma_1\right)\left(\frac{n-1}{4}\right)- \\ \left(\frac{3}{4}F\right)\left(\frac{n^{\frac{5}{3}}-1}{2}\right)+\frac{M^2}{4}\left(\frac{1}{n}-1\right) \\ -\frac{\sigma_1}{12}\left(n^3-1\right)\end{array}\right] \qquad (30)$$

Equation (30) is called the energy integral of an oscillatory particle of mass unity moving with a velocity $n'=dn/d\xi$ at position n in a potential well U(n). It has similar expression as found by Chatterjee *et al.* [61]. If the ion temperature is neglected the equation (30) agrees with equation (19) as reported by Mahmood & Mustaque [56].

The dependence of the Sagdeev's pseudopotential on relativistic degeneracy parameter (F),quantum diffraction parameter(H),and ion- to- electron Fermi temperature ratio( $\sigma$ ) are shown in figures 1 - 6.

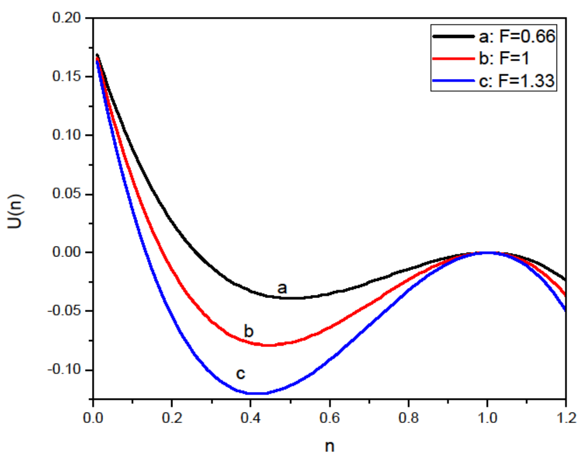


**Fig 1: U(n) is plotted vs. *n* for different values of Relativistic degeneracy parameter F in a weakly relativistic plasma, other parameters are *M*=0.6, *H*=4 and σ=0.1.**

In figure 1 it is found that with cold ions ( $\sigma$ =0.1), the pseudo potential well becomes deeper with increase in F. compared to its ultra-relativistic counterpart it is found that the well are deeper. When the Fermi temperature of the ions is greater than that of electrons the potential wells becomes deeper (fig 2).

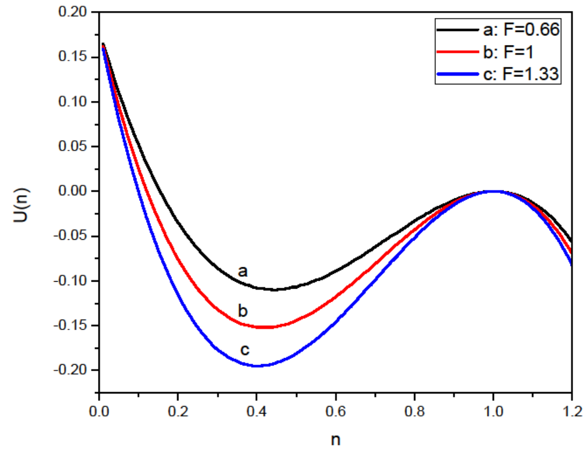


**Fig 2: U(n) is plotted vs. *n* for different values of Relativistic degeneracy parameter F in a weakly relativistic plasma, other parameters are *M*=0.6, *H*=4 and σ=1.2.**

Figures 3 and 4 demonstrates the dependence on quantum diffraction parameter H. It is found that for colder ions the potential well becomes shallower with increase in n. But there is a region of anomaly (0 - $n_m$ ) where the pseudo potential has decreasing value with increasing H.

When compared to its ultra-relativistic counterpart it was further seen that in between the roots of U(n) (i.e. $n_m$ and 1) the well becomes less deep. If ions are compared to the warmer than electron, the depth of the potential well increases (fig 4).

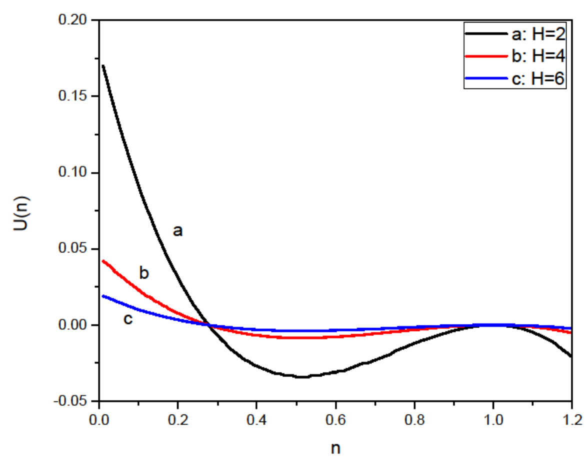


**Fig 3: U(n) is plotted vs. *n* for different values of Quantum diffraction parameter H in a weakly relativistic plasma, other parameters are *M*=0.6, *F*=2/3 and σ=0.1.**

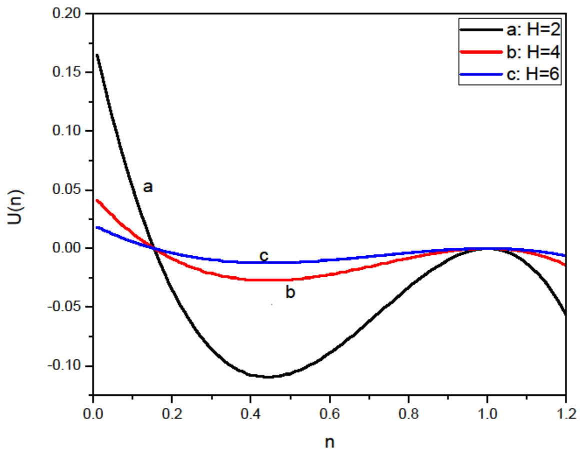


**Fig 4: U(n) is plotted vs. *n* for different values of Quantum diffraction parameter H in a weakly relativistic plasma, other parameters are *M*=0.6, *F*=2/3 and σ=1.2**

The dependence of U(n) on ion temperature is shown in figures 5 & 6. It is found that for colder ions there is very slight variation in U(n) (fig 5) whereas for warmer ions the variation is visible (fig 6).

The dependence of the pseudopotential well on different plasma parameters thus determine the formation of compressive or rarefractive solitary structure or it may even determine the possibility of the formation of double layers

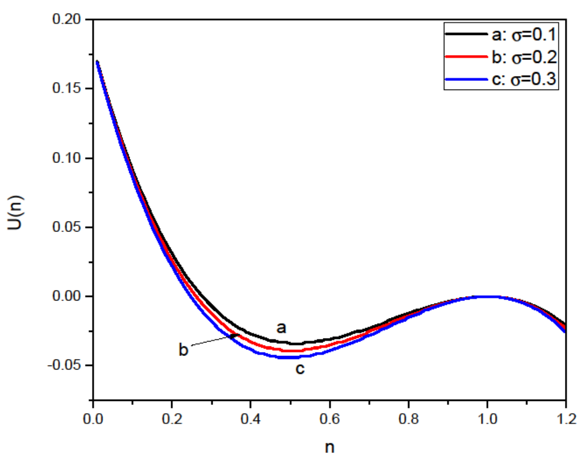


**Fig 5: U(n) is plotted vs. *n* for different values of ion temperature ratio σ in a weakly relativistic plasma, other parameters are *M*=0.6, *F*=2/3 and H=4.**

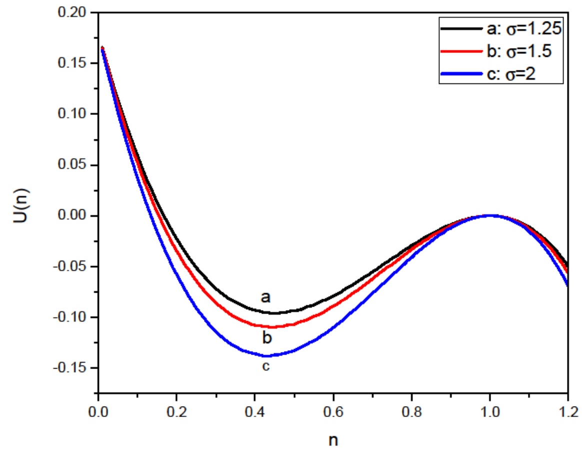


**Fig 6: U(n) is plotted vs. *n* for different values of ion temperature ratio σ in a weakly relativistic plasma, other parameters are *M*=0.6, *F*=2/3 and H=4.**

## III. SOLITARY WAVE SOLUTIONS

The plasma particles motion of a particle whose Sagdeev's pseudopotential well U(n) in *n* is given by Eq. (30). The properties of the pseudopotential *U(n)* will then decide the conditions for the existence of solitary wave solution. If it is found that between any two roots (in this case, 0 and $n_m$) of the pseudopotential, U(n) is negative, then an oscillatory wave is found. On the reverse, if in the interval one root is a single root and another is a double root, then a solitary wave can be predicted [2]. If both the roots are double root, then a double layer exists. The initial conditions are chosen in such a way that the double root appears at *n*=1. Therefore it takes an sufficiently long time to get away from it and *n* reaches zero at $n_m$, then again taking infinitely long time to return to *n*=0. Hence, the conditions for the existence of solitary wave solution are the following:

a) $U(n)=0$ at $n$=1 and $n=n_m$,

b) $\frac{dU(n)}{dn}=0$ at n=1 but $\frac{dU(n)}{dn}\neq 0$ at $n=n_m$ and,

c) $\frac{d^2U(n)}{dn^2}<0$ at n=1

If $n_m$ is less than unity then rarefractive solitary wave structures are formed. On the other hand if it is greater than unity, then compressive structures are obtained. It is to be noted that complex U(n) is not physically allowed as it would imply complex density which is not a physical quantity. From equation (30), it is found that the shape of the solitary structures can be determined from the following:

$$\xi = \pm \int_{n_m}^{n} \frac{dn}{\sqrt{-2U(n)}} \tag{31}$$

The effect of relativistic degeneracy parameter (Figure 7), quantum diffraction parameter (Figure 8) & ion temperature (Figure 9) on the formation and properties of ion acoustic solitons are investigated are investigated.

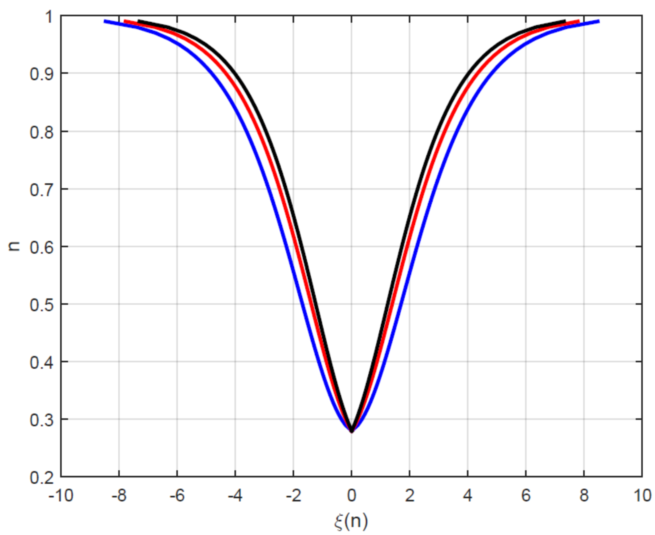


**Fig. 7: *n* is plotted vs. ξ in weakly relativistic plasma with variation of Relativistic Degeneracy parameter F. The blue curve denotes F=2/3, red denotes F=1, black denotes F=4/3; other parameters are *M*=0.6, H=2 and σ=0.1.**

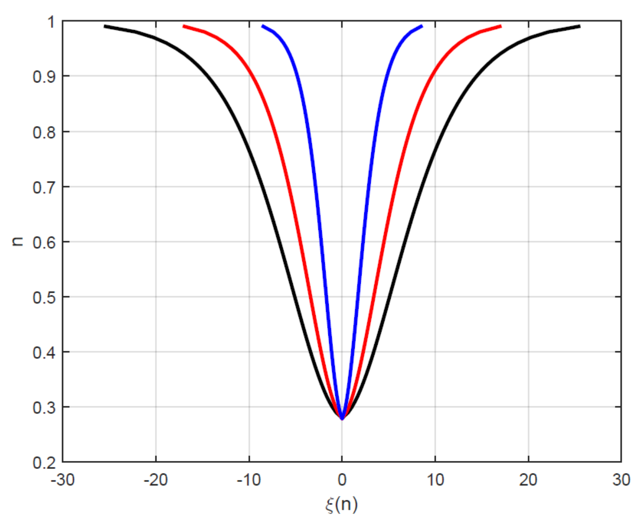


**Fig. 8: *n* is plotted vs. ξ in weakly relativistic plasma with variation of quantum diffraction parameter H. The blue curve denotes H=2, red denotes H=4, black denotes H=6. Other parameters are *M*=0.6, F=2/3 and σ=0.1.**

Figures 7 to 9 shows the dependence of solitary structures in ion acoustic wave in a two-components electron-ion quantum plasma, (in which the ions are colder compare to the electrons) on the relativistic degeneracy parameter(F), quantum diffraction effect(H) & ion to electron Fermi temperature ratio ( $\sigma$ ).

From figure 7 it is found that rarefractive solitons are formed with constant amplitude but with decreasing width (that is become narrower) with increasing value of F. The effect of quantum diffraction effect on ion acoustic solitary structures is shown in figure 8. It is found that the solitary waves become wider with increasing H (a measure of the quantum Bohm potential). The effect of ion-to- electron Fermi temperature ratio ( $\sigma$ ) is shown in figure 9. Here $\sigma$ is varied keeping other parameter constant. It is found that the amplitude remain almost constant, but the width first decreases and then increases with increasing $\sigma$. In the ultra-relative case the picture was slightly different where the amplitude gradually decreases. If ions were consider warmer the similar features were observed but with slightly varying magnitude.

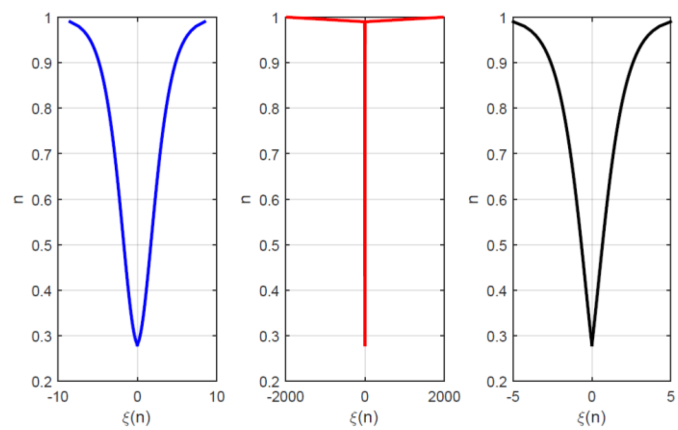


**Fig. 9: *n* is plotted vs. ξ in weakly relativistic plasma with variation of ion temperature ratio σ. The blue curve denotes σ = 0.1, red denotes σ =0.2, black denotes σ = 0.3; other parameters are *M*=0.6, F=2/3 and H=2.**

## IV. CONCLUSION & REMARKS

In these paper arbitrary amplitude solitary structures ion acoustic solitary structures has been investigated in dense quantum plasma containing weakly relativistically degenerate electron & non relativistic ions. The dependency of Sagdeev's pseudopotential well on relativistic degeneracy parameter, quantum diffraction parameter & ion-to electron Fermi temperature are investigated & from this information the feasibility for obtaining ion acoustic solitary structures is investigated. It is found that all these parameters (F, H & $\sigma$ ) significantly affects the properties of ion acoustic solitary waves structures.

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